Ph.D. MATHEMATICS

POOL I Courses:

S.No	Course Code	Title of the Course	L	T	P	C	SE
1	24MA710A	Advance Topics in Algebra	3	0	0	3	100
2	24MA710B	Numerical Methods	3	0	0	3	100
3	24MA710C	Special Functions	3	0	0	3	100
4	24MA710D	Boundary Value Problems	3	0	0	3	100
5	24MA710E	Advanced Linear Algebra (NPTEL)	0	0	3	3	100
6	24MA710F	Numerical Linear Algebra and Its Applications (NPTEL)	0	0	3	3	100
7	24MA710G	Numerical Analysis (NPTEL)	0	0	3	3	100

POOL II Courses:

S.No	Course Code	Title of the Course	L	T	P	C	SE
1	24MA720A	Fuzzy Algebra	3	0	0	3	100
2	24MA720B	Semi Groups	3	0	0	3	100
3	24MA720C	Fluid Dynamics	3	0	0	3	100
4	24MA720D	Fluid Mechanics	3	0	0	3	100
5	24MA720E	Differential Equations	3	0	0	3	100
6	24MA720F	Computational Fluid Dynamics for Incompressible Flows (NPTEL)	0	0	3	3	100
7	24MA720G	Ordinary and Partial Differential Equations and Applications (NPTEL)	0	0	3	3	100

L – Lecture, T – Tutorial, P – Practical, SE – Semester End Exam, C – Credits, Total – Total Marks

POOL I — COURSES SYLLABUS

24MA710A
ADVANCE TOPICS IN ALGEBRA

UNIT-I

Sets: Types of sets, operations on sets.
Function, Matrix: Types of matrices, Determinant of a matrix, Inverse of a matrix, Rank of a matrix.

UNIT-II

Group Theory:
Definition of a Group, Examples of Groups, Preliminary Lemmas, Subgroups, Normal Subgroups and Quotient Groups, Homomorphisms, Automorphisms, Permutation Groups, Cayley's Theorem, Direct Products, Finite Abelian Groups.

UNIT-III

Ring Theory:
Definition and Examples of Rings, Special Classes of Rings, Homomorphisms, Ideals and Quotient Rings, The Field of Quotients of an Integral Domain, Euclidean Rings, Polynomial Rings, Polynomials over the Rational Field, Polynomial Rings over Commutative Rings.

UNIT-IV

Vector Spaces and Modules:
Elementary Basic Concepts, Linear Independence and Bases, Dual Spaces, Inner Product Spaces, Modules.

UNIT-V

Linear Transformations:
The Algebra of Linear Transformations, Characteristic Roots, Canonical Forms: Triangular Form, Canonical Forms: Nilpotent Transformations, Canonical Forms: Jordan Form, Rational Canonical Form, Hermitian, Unitary and Normal Transformations, Real Quadratic Forms.

Text Books

1. L.N. Herstein: *Topics in Algebra*, Second Edition.
2. Rotman, Joseph: *Galois Theory*, Springer 1998.

References

1. Cohn, Paul Moritz: *Classic Algebra*, John Wiley & Sons, 2000.
2. Rotman, Joseph: *Galois Theory*, Springer 1998.
3. Willems, Wolfgang: *Codierungstheorie*; Walter de Gruyter, 1999.

24MA710B
NUMERICAL METHODS

UNIT-I: Numerical Differentiation and Integration

Introduction, Numerical Differentiation, Numerical Integration, Euler-Maclaurin Formula, Adaptive Quadrature Methods, Gaussian Integration, Singular Integrals, Fourier Integrals, Numerical Double Integration.

UNIT-II: Numerical Solution of Ordinary Differential Equations

Introduction, Solution by Taylor's Method, Picard's Method, Euler's Method, Runge-Kutta Methods, Predictor-Corrector Methods, the Cubic Spline Method, Simultaneous and Higher Order Equations, Boundary Value Problems: Finite-Difference Method, The Shooting Method.

UNIT-III: Numerical Solution of Partial Differential Equations

Introduction, Finite-Difference Approximations, Laplace's Equation: Jacobi's Method, Gauss-Seidel Method, SOR Method, ADI Method, Parabolic Equations, Iterative Methods, Hyperbolic Equations.

UNIT-IV: System of Linear Algebraic Equations

Introduction, Solution of Centro-symmetric Equations, Direct Methods, LU-Decomposition Methods, Iterative Methods, Ill-conditioned Linear Systems.

UNIT-V: The Finite Element Method

Functionals, Base Function Methods of Approximation, The Rayleigh-Ritz Method, The Galerkin Method, Application to Two-Dimensional Problems, Finite Element Method for One and Two Dimensional Problems.

Reference Books

1. Niyogi, Pradip: *Numerical Analysis and Algorithms*, Tata McGraw–Hill.
2. Balagurusamy, E.: *Numerical Methods*, Tata McGraw–Hill.
3. Sastry, S.S.: *Introduction to Numerical Analysis*, PHI.
4. Chapra, S.C. and Canale, R.P.: *Numerical Methods for Engineers*, Tata McGraw–Hill.

24MA710C
SPECIAL FUNCTIONS

UNIT-I: The Gamma and Beta Functions

The Gamma function, a series for $\Gamma'(z)/\Gamma(z)$, evaluation of $\Gamma'(1)$, the Euler product for $\Gamma'(z)$, the difference equation $\Gamma(z+1) = z\Gamma(z)$, evaluation of certain infinite products, Euler's integral for $\Gamma(z)$, the Beta function, the value of $\Gamma(z)\Gamma(1-z)$, the factorial function, Legendre's duplication formula, Gauss multiplication theorem, a summation formula due to Euler.

UNIT-II: Bessel Functions

Definition of $J_n(x)$, Bessel's differential equation, differential recurrence relation, a pure recurrence relation, a generating function, Bessel's integral, index half an odd integral, modified Bessel function, orthogonality property for $J_n(x)$.

UNIT-III: Legendre's Polynomials

Definition of $P_n(x)$, differential recurrence relations, the pure recurrence relation, Legendre's differential equation, the Rodrigue's formula, orthogonality property, special properties of $P_n(x)$, more generating functions, Laplace's first integral form, expansion of x^n.

UNIT-IV: Hermite Polynomials

Definition of $H_n(x)$, recurrence relations, the Rodrigue's formula, other generating functions, integrals, the Hermite polynomials as ${}_2F_0$, orthogonality, expansion of polynomials, more generating functions.

UNIT-V: Laguerre Polynomials

The Laguerre polynomial definition, generating functions, recurrence relations, the Rodrigue's formula, the differential equation, orthogonality, expansion of polynomials, special properties, other generating functions, the simple Laguerre polynomials.

Text Book

1. E.D. Rainville: *Special Functions*, MacMillan Company, New York, 1960.

24MA710D
BOUNDARY VALUE PROBLEMS

Eight questions are to be set, and the student has to answer five in three hours of duration.

UNIT-I: System of Linear Differential Equations

System of first-order equations, existence and uniqueness theorem, fundamental matrix, non-homogeneous linear systems, linear systems with constant coefficients.

UNIT-II: Existence and Uniqueness of Solutions

Introduction, preliminaries, successive approximations, Picard's theorem, continuation and dependence on initial condition, existence of solutions in the large interval. (Scope and treatment as in Chapters 4 and 5 of Text Book (1)).

UNIT-III: Nonlinear Boundary Value Problems

Kinds of boundary value problems associated with nonlinear second-order differential equations, generalized Lipschitz condition, failure of existence and uniqueness of linear boundary value problems, simple nonlinear BVP, standard results concerning initial value problems.

UNIT-IV: Relation Between the First and Second Boundary Value Problems

Relation between uniqueness intervals, relation between existence intervals.

UNIT-V: Contraction Mapping

Introduction, contraction mappings, boundary value problems, a more generalized Lipschitz condition. (Scope and treatment as in Chapters 1, 2, and Sections 3.1 to 3.4 of Chapter 3 of Text Book (2)).

Text Books

1. S.G. Deo, V. Lakshmikantham, and V. Raghavendra: *Textbook of Ordinary Differential Equations*, Second Edition, Tata McGraw-Hill Publishing Company Ltd, New Delhi (2002).

2. P.B. Bailey, L.P. Shampine, and P.E. Waltman: *Non-linear Two Point Boundary Value Problems*, Academic Press, New York and London (1968).

POOL II — COURSES

SYLLABUS

24MA720A
Fuzzy Algebra

Unit-I: Fuzzy Subsets & Fuzzy Subgroups

Union of two fuzzy subgroups, fuzzy subgroup generated by a fuzzy subset, fuzzy normal subgroups, fuzzy conjugate subgroups and fuzzy characteristic subgroups, fuzzy Sylow subgroups.

Unit-II: Fuzzy Subrings and Fuzzy Ideals

Basic concepts, properties of fuzzy ideals, union of fuzzy subrings (fuzzy ideals), fuzzy subring (fuzzy ideal) generated by a fuzzy subset, fuzzy ideals and homomorphism, fuzzy cosets.

Unit-III: Fuzzy Prime Ideals and Maximal Ideals

Fuzzy prime ideals, fuzzy maximal ideals, fuzzy semi-prime ideals, characterization of regularity.

Unit-IV: Fuzzy Primary Ideals

Fuzzy primary ideals, fuzzy semi-primary ideals: definition and some properties, fuzzy ideals and irreducible ideals in Noetherian rings.

Notes:

1. Eight questions are to be set, out of which five questions are to be answered. 2. Questions should be uniformly distributed across all the units.

Prescribed Textbook:

Fuzzy Algebra by Rajesh Kumar, University Press, University of Delhi, Delhi-110007.

Reference Textbook:

Fuzzy Commutative Algebra by John N. Mordeson & D. S. Malik, World Scientific Publishing Co. Pte. Ltd.

24MA720B
Semi Groups

Unit-I: Functions on a Semigroup

Semigroup, special subsets of a semigroup, special elements of a semigroup, relation and functions on a semigroup, transformations, free semigroups.

Unit-II: Ideals and Related Concepts

Subdirect products, completing prime ideals and filters, completely semiprime ideals, semilattices of simple semigroups, weekly commutative semigroups, separative semigroups, Γ-semigroups.

Unit-III: Ideal Extensions

Extensions and translations, extensions of a weekly reductive semigroup, strict and pure extensions, retract extensions, dense extensions, extensions of an arbitrary semigroups, semilattice compositions.

Unit-IV: Completely Regular Semigroups

Completely regular, completely simple semigroups, semilattices of rectangular groups, strong semilattice of completely simple semigroups, subdirect product of a semilattice and a completely simple semigroup.

Unit-V: Inverse Semigroups

The natural partial order of an inverse semigroup, partial right congruences on an inverse semigroup, representations by one-to-one partial transformations, homomorphisms of inverse semigroups, semilattices of inverse semigroups.

Note:

1. 8 Questions to be set, out of which 5 Questions to be answered.
2. Questions should be uniformly distributed across all the units.

Prescribed Textbook:

1. *Introduction to Semigroups* by Mario Petrich; Charles E. Merrill Publishing Company.
2. *The Algebraic Theory of Semigroups Volume II* by A.H. Clifford and G.B. Preston; American Mathematical Society.

Reference Textbook:

1. *The Algebraic Theory of Semigroups* by A.H. Clifford and G.B. Preston; American Mathematical Society, First Edition.

24MA720C
Fluid Dynamics

Unit-I: Kinematics of Fluids in Motion

Real fluids and ideal fluids, velocity of a fluid at a point, streamlines, path lines, steady and unsteady flows, velocity potential, the vorticity vector, local and particle rates of changes, equations of continuity, worked examples, acceleration of a fluid, conditions at a rigid boundary.

Unit-II: Equations of Motion of a Fluid

Pressure at a point in a fluid at rest, pressure at a point in a moving fluid, conditions at a boundary of two inviscid immiscible fluids, Euler's equation of motion, discussion of the case of steady motion under conservative body forces.

Unit-III: Some Three-Dimensional Flows

Introduction, sources, ranks and doublets, images in a rigid infinite plane, axis symmetric flows, Stokes stream function.

Unit-IV: Some Two-Dimensional Flows

Meaning of two-dimensional flow, use of cylindrical polar coordinates, the stream function, the complex potential for two-dimensional, irrotational incompressible flow, complex velocity potentials for standard two-dimensional flows, some worked examples, two-dimensional image systems, the Milne-Thompson circle theorem.

Unit-V: Viscous Flows

Stress components in a real fluid, relations between Cartesian components of stress, translational motion of fluid element, the rate of strain quadric and principle stresses, some further properties of the rate of strain quadric, stress analysis in fluid motion, relation between stress and rate of strain, the coefficient of viscosity and laminar flow, the Navier-Stokes equations of motion of a viscous fluid.

Contents:

F. Chorlton, *Text Book of Fluid Dynamics*, CBS Publications, Delhi, 1985.
Unit 1: Chapter 2, Sec 2.1 to 2.10.
Unit 2: Chapter 3, Sec 3.1 to 3.7.
Unit 3: Chapter 4, Sec 4.1 to 4.5.
Unit 4: Chapter 5, Sec 5.1 to 5.8.
Unit 5: Chapter 8, Sec 8.1 to 8.9.

References: 1. G.K. Batchelor, *An Introduction to Fluid Mechanics*, Foundation Books, New Delhi, 1984.
2. A.J. Chorin and A. Marsden, *A Mathematical Introduction to Fluid Dynamics*, Springer Verlag, New York, 1993.
3. S.W. Yuan, *Foundations of Fluid Mechanics*, Prentice Hall of India Pvt. Ltd., New Delhi, 1976.
4. R.K. Rathy, *An Introduction to Fluid Dynamics*, Oxford and IBH Publishing Company, New Delhi, 1976.

24MA720D
Fluid Mechanics

Unit-I: Basics in Fluid Mechanics

The continuum hypothesis, Newtonian and Non-Newtonian fluids, continuity equation, Navier-Stokes equations of motion, energy equation, steady and unsteady flows.

Unit-II: Navier-Stokes Equations

Parallel flow through a straight channel and Couette flow, the Hagen-Poiseuille flow, the suddenly accelerated plane wall, Stokes first problem, the flow near an oscillating flat plate (Stokes second problem), flow near a rotating disk, parallel flow past a sphere.

Unit-III: Boundary Layer Theory

Derivation of boundary layer equations for two-dimensional flow, the separation of a boundary layer, skin friction, the boundary layer on a flat plate.

Unit-IV: Thermal Boundary Layers in Laminar Flow

Exact solution for the problem of temperature distribution in a viscous flow: (i) Couette flow, (ii) Poiseuille flow through a channel with flat walls. Forced and natural flows, thermal boundary layer in forced flow, parallel flow past a flat plate at zero incidence, thermal boundary layers in natural flow (free convection).

Textbooks:

1. *Boundary Layer Theory* by Dr. Herman Schlichting, McGraw Hill Book Company.
2. *Fluid Mechanics and Fluid Machines* by S.K. Som & G. Biswas.

Reference Books:

1. F. Chorlton, *Textbook of Fluid Dynamics*, Van Nostrand, 1963.

24MA720E
Differential Equations

Unit-I: System of Linear Differential Equations

System of first-order equations, existence and uniqueness theorem, fundamental matrix, non-homogeneous linear system, linear systems with constant coefficients.

Unit-II: Existence and Uniqueness of Solutions

Introduction, preliminaries, successive approximations, Picard's theorem, continuation and dependence on initial condition, existence of solutions in the large interval.

(Scope and treatment as in Chapters: 4 and 5 of Textbook (1)).

Unit-III: Oscillation Theory and Boundary Value Problems

Qualitative properties of solutions, the Sturm comparison theorem, Eigenvalues, Eigenfunctions.

Unit-IV: Power Series Solutions and Special Functions

Series solutions of first-order and second-order linear differential equations, ordinary points, regular singular points. Gauss's hypergeometric equation, the point at infinity.

Unit-V: Non-Linear Equations

Autonomous systems, the phase plane and its phenomena, type of critical points, stability, critical points and stability for linear systems, stability by Liapunov's direct method, simple critical points of non-linear systems.

(Scope and treatment as in Chapters: 4, 5 (Sections 25-29), and 8 (Sections 40-44) of Textbook (2)).

Text Books

1. *Textbook of Ordinary Differential Equations* by S.G. Deo, V. Lakshmikantham, and V. Raghavendra, Second Edition, Tata McGraw-Hill Publishing Company Ltd., New Delhi, 2002.

2. *Differential Equations with Applications and Historical Notes* by George F. Simmons, Tata McGraw-Hill Publishing Company Ltd., New Delhi, 1972.